Status report for ECT image reconstruction

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Chapitre 1

Main goal: Precise Image Reconstruction

The Electrical Capacitance Tomography (ECT) has been chosen as probing technique for the experimental investigation of plugging of trickle-bed reactors (and the use of induced pulsing to counter plugging).

The main characteristics of ECT are listed below:

- it is based on dielectric properties of materials (well suited for chemical processes);
- it is non-intrusive (electrodes placed on the outside of the reactor wall).

ECT has a number of important advantages when compared with other methods of non-intrusive investigation in chemical reactions. First and most important, ECT is fast. It is possible to record around 100 frames (images) per second (compared with one frame per 1 to 30 seconds for other types of tomography). Still, it is not as expensive as hard field tomography. Also very important, ECT is essentially harmless for human operators and environment.

There are a number of disadvantages too, in ECT. The nature of the measuring electric field is embarrassingly non-linear. This introduces very much indetermination in the reconstruction of the tomograms. The reconstruction itself implies the resolution of an inverse problem of difficult nature, mathematically and numerically.

In the time period covered by the present report, the section of the project concerned with numerical and computing aspects has focused on improving the quality of the image reconstruction, in order to assure the maximum possible precision and reliability in the interpretation of tomography data.

The high precision requirements come from the nature of the studied reactor setup. A four
phase trickle bed reactor contains the following main components:

- the catalytic bed formed of porous catalyst (in our experiments γ-alumina) pellets;
- the liquid phase (in our experiments kerosene) flowing downwards through the catalytic bed;
- the gas phase (reactant, in our experiments air) flowing in cocurrent;
- the solid fines (in our experiments kaolin) in suspension in the liquid phase.

It is the solid suspension that produces the plugging by forming deposits in the interstices of the catalytic bed. It was initially supposed that the geometric scale of this phenomenon was very near the resolution of ECT and this explains why the main goal of this section of the project was fixed to the search for precise reconstruction methods for the ECT images.

The software that accompanies the ECT installation is lacking in what concerns high quality image reconstruction algorithms. It was thus necessary to find in literature (or discover) and implement such algorithms in the form of computer programs that be applied on the tomography capacitance data in order to recover the maximum of information with most possible precision.

1.1 Literature Study

An extensive literature study (see bibliography) revealed that a great attention has been shown to the improvement of image reconstruction algorithms in ECT. The specificities of this hard problem are related with those of other popular research topics like generic image reconstruction, automatic image analysis and resolution of mathematically ill-defined inverse problems.

The fundamental transfer equation for the electrical conduction in a planar section is given below:

\[
\nabla (\varepsilon_0 \varepsilon(x,y) \nabla \phi(x,y)) = 0
\]

From this, the capacitance between two specific electrodes is deduced [Yang et Peng, 2003]:

\[
C = -\frac{1}{\Delta V} \int \int_{\Gamma} \varepsilon_0 \varepsilon(x,y) \nabla \phi(x,y) d\Gamma \quad \text{or, generally:} \quad C = \xi(\varepsilon)
\]

where \(C\) is the capacitance, \(\Delta V\) represents the difference of potential between electrodes, \(\varepsilon\) means permittivity, \(\phi\) is electrical potential and \(\Gamma\) symbolises external surface of electrode.
By derivation, we obtain:

$$\Delta C = \frac{d\varepsilon}{d\varepsilon} (\Delta \varepsilon) + O^2(\Delta \varepsilon) \quad \text{or} \quad \Delta C = S \Delta \varepsilon \quad (1.3)$$

where $S = \frac{d\varepsilon}{d\varepsilon}$ is an operator that indicates the sensibility of the capacitance to the variations of the permittivity in the cross-section of the ECT sensor.

After normalisation, results:

$$\lambda = S g \quad (1.4)$$

where $\lambda$ represents the vector of normalised capacitances (length $M$), $g$ is permittivities vector (length $N$) and $S$ represents the sensibility matrix (of dimension $M \times N$).

We can derive from the above the inverse problem to be solved: determine $g$ knowing $S$ (measured experimentally) and given $\lambda$ (measured for the unknown $g$):

$$g = S^{-1} \lambda \quad (1.5)$$

It is immediately noticed that the problem is critically ill-posed: $M$ is much larger than $N$ thus $S$ is non-inversible. This means that the equation (1.5) has a large set of equally plausible solutions, from which it is impossible to unequivocally distinguish the unique actual solution.

The literature study revealed two fundamental categories of algorithms aimed at solving this inverse problem: classic algorithms, subdivised in non-iterative and iterative, and neural network algorithms [Yang et Peng, 2003; Øyvind Isaksen, 1996; Warsito et Fan, 2003c].

### 1.2 Classic Algorithms

The **Linear Back Projection (LBP)** is the most largely used image reconstruction algorithm for ECT is borrowed from the hard field tomography. The fundamental equation of this algorithm is:

$$\hat{g} = \frac{S^T \lambda}{S^T u_k} \quad (1.6)$$

The main advantage of LBP is its simplicity, which also makes it easy to perform by a computer. Thus reconstruction with this algorithm is very fast. The downside consists in the really poor
quality of the reconstructed image. An impairing smoothing/smearing occurs in the reconstructed image when compared with the real image.

The Single Value Decomposition (SVD) (and its truncated version) algorithms are mathematically described as:

\[ S = U \Sigma V^T \quad \text{then} \quad \hat{g} = V \Sigma^{-1} U^T \lambda \]  

(1.7)

They offer very interesting image quality for the low computing costs that it implies.

The Tikhonov regularisation method [Tikhonov et Arsenin, 1976] means:

\[ \hat{g} = (S^T S + \mu I)^{-1} S^T \lambda \]  

(1.8)

This method seems to be difficult to control and requires arbitrarily chosen parameters (\( \mu \)).

The Iterative Tikhonov method performs, at each iteration, a Tikhonov regularisation:

\[ \hat{g}_{k+1} = \hat{g}_k - (S^T S + \mu I)^{-1} S^T (S \hat{g}_k - \lambda) \]  

(1.9)

The Landweber and Projected Landweber methods [Yang et al., 1999; Liang et Xu, 2003] are iterative algorithms inspired from the photographic image repairing research.

\[ \hat{g}_{k+1} = \hat{g}_k - \alpha S^T (S \hat{g}_k - \lambda) \]  

(1.10)

Until very recently, the Landweber algorithm was considered the “state of the art” in ECT image reconstruction. Unfortunately it has rather severe numerical inconveniences, related to the necessity of using a number of arbitrary parameters as well as lacking a reliable iteration stop criterium.

The Simultaneous Iterative Reconstruction Technique (SIRT) is also inspired from the hard field tomography.

\[ \hat{g}_{k+1} = \hat{g}_k - \beta S^T \frac{S \hat{g}_k - \lambda}{\text{diag}SS^T} \]  

(1.11)

It is very similar to the Landweber method. Its main advantage resides in the way it is often implemented: \( \beta \) is a weighting vector (allowing to give discriminate importance to specific parts of the measured capacitances vector).
We implemented all these classic algorithms in a programming framework based on the Python programming language and using Python’s high performance numerical libraries. We used these implementations for comparative studies. A graphical comparison of simulation results obtained from these studies is given at figures 1.1 and 1.2.

We can observe in the figures 1.1 and 1.2 how different image reconstruction algorithms behave given a particular original image (phantom). One essential observation is immediate: images of distinct objects (like spots, bars and multiple spots in figure 1.1) are relatively well reconstructed; when it comes to anular or quadratic profiles (like those shown in 1.2), the reconstructed images make almost impossible to differentiate importantly different original permit- tivity setups. The most disturbing is the difficulty to distinguish between anular and parabolic setups as given in row 1 and row 4 of figure 1.2. Such situations are almost equally possible to occur in a plugging trickle bed reactor and thus the imaging technology we choose should enable us to differentiate them.

1.3 Neural Network Algorithms

Warsito et Fan [2001] have started very recently to develop (and to propose) an image reconstruction technique based on the use of neural networks and which takes advantage of the principles of multicriterium optimisation [Warsito et Fan, 2003b,c,a; Du et al., 2004].

According to the cited authors, the technique is very promising, mostly thanks to its reconstruction accuracy. The main disadvantage consists in its extensive use of computing time, which makes it misfit for real-time image reconstruction, in synchronisation with the image aquisition frequency of the electrical capacitance tomograph.

We started developing a similar image reconstruction algorithm based on Hopfield neural netowrks. The work is in progress and the first usable results are expected during the month of June 2005. The advance is hindered by the probably intentional ambiguity of the literature concerning different arbitrary parameters and numerical conditions that the original authors use in their actual computer codes.
Figure 1.1: Comparative results of classic image reconstruction algorithms (discernable patterns)

Figure 1.2: Comparative results of classic image reconstruction algorithms (problematic patterns)
Chapitre 2

Sources of Capacitance Data for Image Reconstruction

In order to test and ultimately use the numerical algorithms developed for ECT image reconstruction, we need sources of tomography data in the form of electrical capacitances.

While for the final stages of our research we would need actual tomograms as provided by the electrical capacitance tomograph, for the algorithm test stages we needed to have a control over the original permittivity image from which capacitances be somehow obtained, which can then be fed to reconstruction algorithms. The reconstruction images then would need visual and numerical comparison with the original image, in order to classify the different algorithms based on their quality and precision.

Thus, for the needs of algorithm testing, we decided to use a finite element simulation code library named MEF++ and developed by the GIREF research group from Laval University in Québec City.

2.1 Finite Element Simulations

MEF++ is a very versatile library that enables a numerical researcher to build mathematical solvers for a large variety of differential equations with partial derivatives (like e.g. (1.1)).

The current project needed such a tool in order to derive simulated capacitance data starting from perfectly characterised permittivity images.
2.1.1 Building the FE Simulator

MEF++ needed a few adaptations in order to be useful for the numerical resolution of equation (1.1).

The integration of this equation, as well as the integration of the capacitances on the geometrical representation of the sensor’s electrodes implies:

- integrating on 1D entities (lines) inside a 2D domain when simulating a planar cross section of the capacitance sensor;
- integrating the electric field on 2D entities (surfaces) inside a 3D domain when simulating the whole cylindrical volume of the capacitance sensor.

These implications imposed the development of special numerical techniques which didn’t exist in MEF++ previously (and probably don’t exist in most commercial or academic Finite Element Method (FEM) libraries).

Currently, we detain a very robust simulator able to provide simulated capacitance data derived from arbitrarily complicated permittivity images.

A variant of this simulator is used for computing sensitivity matrices (S), an essential parameter of the reconstruction algorithms. For obtaining a sensibility matrice, one has to fix successively each pixel of the image to the highest used permittivity while leaving the rest of the image at the lowest permittivity and thus compute the associated capacitance data series for each pixel. This operation can be performed physically, in the tomograph, but this would require a very large amount of work and time. Performing this operation by numerical simulation insures a high precision in the computations as well as reproductibility.

2.1.2 Preparation of Meshes with GMSH

A FEM simulator has a few essential parts that are instrumental in its correct functioning. The most important is the Finite Element formulation, which has to be mathematically exact and most of the times needs to be augmented with specific (and special) numerical algorithms for solving numerically hard algorithmic difficulties.

The other essential part of a FEM simulator is the mesh through which is indicated the discretisation of the domain on which the solution has to be integrated. There are a few important aspects in the conception of the discretisation mesh:

1The 3D variant of the simulator is a work in progress. Yet, given that the tridimensional simulator is mainly a generalisation of the planar simulator, the most of the work required for creating the specific algorithms is already done.
• precise representation of the geometry of the domain;
• appropriate discretisation of the domain, depending on the solution to integrate;
• proper delimitation and marking of the entities in the domain which will have to satisfy
  initial and boundary conditions;

The implementation of our FEM simulator required the devising of two categories of meshes:

• non-structured meshes for capacitance data simulations;
• partially structured meshes for the computation of sensitivity maps and for the image
  reconstruction;

We used the GMSH mesh generator, a high quality free tool available from
http://geuz.org/gmsh. The figures 2.1 and 2.2 are showing examples of the meshes cre-
ated and used for this project. The partially structured grid requires particularly large amounts
of time and manual work in order to generate it properly.

Figure 2.1: Non-structured grid used for the FE simulation of capacitance data
2.2 Tomography

The tomography recordings are done with an electrical capacitance tomograph from Process Tomography Limited (PTL). The sensor of this tomograph is connected to a sophisticated electronic controller. This controller is able to drive the electrical potentials required for the capacitance measurements (see figure 2.3). It also has the task to transfer to a computer the capacitance data read from the sensor.

The computing system provided by PTL comes with a dedicated software which interfaces the user with the electronic controller. This software has also some limited abilities of image reconstruction, based on the simplest non-iterative algorithms (see section 1.2).

We thus have to extract the capacitance data from this software and use it with our own set of image reconstruction algorithms. Unfortunately, the data can only be extracted in the form of a data file having proprietary binary formats. We needed to write special programs for the decrypting and translation of these data files.
2.2.1 Decrypting Capacitance Files

The PTL tomograph provides capacitance data in form of .BCP files, containing series of measurements, each element of the series representing the capacitance data for one sensor crosssection field detection.

Decrypting the .BCP files proved relatively easy to do. We were able to extract the capacitance series, but we discovered that the data in the series wasn’t given following the ordered positions of the electrodes in the sensors.

With a sensor consisting of 12 electrodes, the tomograph can perform 66 $(N \times (N-1)/2)$ independent measurements. The representation of the capacitance values of such a series of 66 measurements reported to the order of electrode positions should give a graph like the one shown in figure 2.2.2

2.2.2 Decrypting Sensitivity Files

The key to the correct ordering of capacity data in .BCP files proved to be encrypted in the sensitivity map files that have to be used with the PTL control software. These files, with the extension .SIF, contain sensor geometry information, permittivity range information, electrode signal ordering information and sensitivity maps.

The decrypting of this type of files is work in progress and should be completed in during the month of May 2005.
Figure 2.4: Representation of capacitance values reported to the order of electrode positions in the ECT sensor

Figure 2.5: Representation of capacitance values as extracted from a .BCP file
Chapitre 3

Difficulties with Tomography of Four Component Systems

The ECT was initially devised for the imaging of two-phase systems, i.e. imaging of structures consisting of two materials (or components) with distinct electrical permittivities $\varepsilon_1$ and $\varepsilon_2$.

In other words, ECT can detect images of objects of high permittivity (e.g. organic liquid) inside a space filled with low permittivity material (e.g. gas like air or azot). The reciprocal setup can also be used (objects of low permittivity in environments of high permittivity) but with some limitations.

Another restriction of ECT is that the objects to be imaged have to be larger than the resolution of the tomography sensor, which is most often one thirty-th ($1/30$) of the internal diameter of the sensor.

The literature [see Warsito et Fan, 2003b] presents methodologies for using ECT on three-phase systems. The very simplified description of such methodologies is that, by mathematical manipulation, the software used to reconstruct the tomograms uses the differences of permittivities $\Delta\varepsilon_{a,b} = \varepsilon_b - \varepsilon_a$ and $\Delta\varepsilon_{b,c} = \varepsilon_c - \varepsilon_b$ as measurement variables instead of $\varepsilon_1$ and $\varepsilon_2$.

In the case of four-phase trickle bed reactors, we need to do imaging of a four-phase system. In fact, because of complications with maintaining a stable suspension of the kaolin fines in the liquid phase, it was necessary to add a surfactant to the mixture of fines with the liquid phase (cetyl-tri-methyl-amonium bromide (CTAB) was used).

Even though it represents only 5% weight of the mass of the solid kaolin fines suspended in the system, the surfactant has a relative permittivity 7 to 8 times larger than that of the files. The surfactant is meant to cover entirely the particles of kaolin in order to help with the stabilisation
of their suspension in the liquid phase. But this also drastically changes the behavior of the fines in the electric field, as the surfactant coating largely modifies their apparent permittivity.

The phases, with their respective average permittivities are:

<table>
<thead>
<tr>
<th>Phase</th>
<th>Permittivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>solid phase (catalytic bed, γ-alumina)</td>
<td>$\varepsilon_s = 4.5 \times \varepsilon_0$</td>
</tr>
<tr>
<td>liquid phase (kerosene)</td>
<td>$\varepsilon_l = 2.2 \times \varepsilon_0$</td>
</tr>
<tr>
<td>gas phase (air)</td>
<td>$\varepsilon_g \approx 1.0 \times \varepsilon_0$</td>
</tr>
<tr>
<td>suspension of solid fines (kaolin)</td>
<td>$\varepsilon_f = 5.0 \times \varepsilon_0$</td>
</tr>
<tr>
<td>surfactant (CTAB)</td>
<td>$\varepsilon_f = 36.0 \times \varepsilon_0$</td>
</tr>
</tbody>
</table>

$\varepsilon_0 = 8.8524 \times 10^{-12} \text{ F/m}$ is the absolute permittivity of vacuum.

Given the integrative nature the physical phenomena used to determine the value of the electric field that goes through the sensor in ECT, having four different materials inside the investigated trickle bed reactors has the effect of strongly increasing the degree of indetermination the image reconstruction algorithms have to cope with.

We’re presently in debate about what instrumental or algorithmic techniques could help to reduce this degree of indetermination. The theory wants that doing a larger number of independent measurement should do exactly this. It is difficult to devise ways for increasing the number of independent measurements. We have considered:

- fast alternations of different intensity electric fields;
- temperature variation;
- alternative alteration of the permittivity of one of the phases (like injecting a pulse of higher permittivity component in the liquid phase);

Each of these research venues shows upfront important inconveniences, the most important always being the increase of the number of unknowns in the experimental system.
Chapitre 4

Future Developments

We are presently at the stage of collecting a large database of capacitance data series taken on a functional pilot trickle bed reactor at different stages of plugging.

We have settled at using the Projected Landweber algorithm for all the image reconstructions that we do from this capacitance series database. We are currently considering the creation of an expert-system algorithm that would help to automatically differentiate reconstructed images of distinct stages of plugging. We are aware that the biggest difficulty will be to accurately diagnose preferential flow path formation at the wall of the reactor, since ECT will represent such an occurrence the same as a parabolic plug flow. We are reserving our analysis efforts for after the moment the database will be complete.

Another important direction is the creation of a Hopfield Neural Network image reconstruction algorithm in order to compare its performances (when applied to trickle bed reactor investigation) with those of the best classical algorithm, the Projected Landweber.

The decryption of the sensitivity map files will be completed shortly.

Following hints from discussions with industrial partners, we intend to perform a number of numerical simulations in order to investigate the usability of inside-wall ECT sensors in gas holdup measurements.
Chapitre 5

Conclusions

Image reconstruction for ECT is a very difficult problem to solve with acceptable precision and reliability. We compared a number of image reconstruction algorithms as described in literature and we concluded that only marginal improvements could be brought to these algorithms, while the work and time investment would be substantial.

Using ECT for four (or even five) -phase systems, like the four-phase trickle reaction, further complicates the already difficult problem of image reconstruction.

ECT is an excellent diagnostic tool thanks to its high speed and its being harmless to human operators. Yet using it for precision measurements and imaging of trickle beds proves to be very challenging.
Bibliographie


